Evaluating Dimensional Distinctness with Correlated-Factor Models: Limitations and Suggestions

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Abstract

Many differential cognitive psychologists appear to interpret correlated-factor models associated with inter-latent variable correlations meaningfully less than 1.0 as support for the plausibility of several related, but to some degree distinct, dimensions. It is contended in this paper that such a conclusion drawn from a well-fitting correlated-factor model may not be justified necessarily, even if the correlated-factor model fits better than a single-factor model. Based on a series of simulated correlation matrices, it is demonstrated that a well-fitting correlated-factor model with inter-latent variable correlations less than 1.0 only suggests the possibility of several distinct group-level dimensions. In order to obtain additional useful information relevant to the distinctness of the hypothesized group-level factors, a higher-order model is demonstrated to be particularly useful, especially when complemented with omega hierarchical subscale (omegaHS), an effect size index of unique latent variable strength. The following guidelines are provided to help interpret the magnitude of omegaHS values: relatively small < .20; typical .20 to .30; and relatively large > .30. The implications of the simulation are demonstrated based on the re-analysis of three previously published correlations relevant to cognitive processes. Researchers are encouraged to supplement correlated-factor model analyses with higher-order model analyses, in order to evaluate the distinctness of the hypothesized dimensions more fully.

Keywords: confirmatory factor analysis, correlated-factor model, higher-order model, reliability, factorial validity

Evaluating Dimensional Distinctness with Correlated-Factor Models: Limitations and Suggestions

Researchers have been recommended to use a competing models strategy in the application of confirmatory factor analysis (Jöreskog, 1993). A commonly observed and recommended model comparison is that between a single-factor model and a correlated-factor model (Kline, 2011; Brown, 2015; Byrne, 2010; Zeller & Carmines, 1980). In practice, differential cognitive psychologists who endorse a correlated-factor model acknowledge the association between the latent variables. However, they also tend to make statements relevant to the separability, or uniqueness, of each of the hypothesized specific dimensions. For example, McAuley and White (2011) reported a well-fitting correlated three-factor model with inter-latent variable correlations less than 1.0 and concluded that: "This model is consistent with the hypothesis that processing speed, response inhibition, and working memory are separable abilities" (p. 461). In another investigation, Neubert, Kretzschmar, Wüstenberg, and Greiff (2015) endorsed a well-fitting correlated-factor model from the perspective that the latent variables represented "…strongly related, but nonetheless separable dimensions of CPS [complex problem solving] …" (p. 186). Finally, Miyake et al. (2000) stated that the "… three target functions (i.e., Shifting, Updating, and Inhibition) are clearly distinguishable … [but] do seem to share some underlying commonality" (pp. 86-87), based on a well-fitting correlated three-factor model with inter-latent variable correlations less than 1.0. There are many similar examples in the cognitive literature (e.g., Alloway, Gathercole, & Pickering, 2006; Deary, McCrimmon, & Bradshaw, 1997; Engel de Abreu, Conway, & Gathercole, 2010; Friedman et al., 2006; Giofrè, Mammarella, & Cornoldi, 2013; Greiff et al., 2013; Hegarty, 2004; Hicks, Harrison, & Engle, 2015; Janssen, De Boeck, & Vander Steene, 1996; Kail & Hall, 2001;

Mackintosh & Bennett, 2003; Miyake, Friedman, Rettinger, Shah, & Hegarty, 2001; Santos et al., 2015; Shelton, Elliott, Matthews, Hill, & Gouvier, 2010; Unsworth, Spillers, & Brewer, 2009). In our view, such statements about evidence in favour of the interpretation of specific, unique dimensions are ambiguous, if not unjustified, without additional testing.

In the first part this paper, it will be demonstrated via simulation that a well-fitting correlated three-factor model that fits better than a single-factor model is not necessarily indicative of the plausibility of three or more unique, group-level dimensions, even when the inter-latent variable correlations are substantially less than 1.0. Instead, in order to evaluate clearly the proposition that there are a specified number of specific, group-level dimensions within the data, researchers are encouraged to conduct supplementary model testing; specifically, the higher-order model with emphasis placed on the first-order factor residuals, in addition to omega hierarchical subscale estimates (omegaHS; ω_{hs}).

In the second part this paper, the limitations associated with the correlated-factor model will be demonstrated based on the re-analysis of correlation matrices reported in three previously published confirmatory factor analytic investigations in the area of differential cognitive psychology. To foreshadow the results, it will be shown that a well-fitting correlated-factor model with inter-latent variable correlations less than 1.0 may, or may not, be associated with as many unique group-level dimensions as suggested by the authors. Consequently, it will be argued in this paper that the results associated with a correlated-factor model should not be relied upon, on their own, for the purposes of evaluating what appears to be a typically observed interpretation of a correlated-factor model in the literature.

Single-Factor versus Correlated-Factor Models: Described

The single-factor model is one of simplest models that can be specified to account for the shared variance between indicators (Kline, 2011; Rindskopf & Rose, 1988; Spearman, 1904). As can be seen in Figure 1, Model 1 depicts a single-factor model with one latent variable defined by nine indicators. Theoretically, the single-factor model implies one general construct (g), the presence of which, theoretically, causes the indicators to inter-correlate with each other. In this case, the single-factor model is associated with 27 degrees of freedom.

Model 2 is an example of a restricted correlated three-factor model. There are three latent variables, A, B and C, each defined by three indicators. Furthermore, there is a covariance term between all three latent variables to represent their inter-association. The correlated-factor model is a more complex model than the single-factor model (Rindskopf & Rose, 1988). Correspondingly, in this case, the restricted correlated three-factor model is associated with three fewer degrees of freedom (i.e., 24). Theoretically, the variance common to all indicators in a correlated three-factor model is implied to be due to the shared variance between the three latent variables (a.k.a., group-level factors), rather than a general or global process. The single-factor model and the correlated three-factor model are nested within each other, as the correlated-factor model can recover a single-factor model with three fixed parameters, i.e., the inter-latent variable correlations fixed to 1.0.

In practice, some differential cognitive psychologists appear to interpret well-fitting correlated-factor models, with inter-latent variable correlations less than 1.0, from the perspective that the latent variables represent, to some degree, distinct or specific dimensions (e.g., Adrover-Roig, Sesé, Barceló, & Palmer, 2012; Alloway et al., 2006; Deary et al., 1997; Engel de Abreu et al., 2010; Friedman et al., 2006; Giofrè et al., 2013; Greiff et al., 2013;

Hegarty, 2004; Hicks et al., 2015; Janssen et al., 1996; Kail & Hall, 2001; Mackintosh & Bennett, 2003; McAuley & White, 2011; Miyake et al., 2001; Santos et al., 2015; Shelton et al., 2010; Swanson, Orosco, & Lussier, 2015; Unsworth et al., 2009). However, in order to support the notion that there are three or more distinct dimensions within a correlated-factor model, it is suggested in this paper that there should be some unique (unshared) true score variance associated with *each* of the postulated group-level latent variables/dimensions. In practice, it is difficult to determine whether there is, in fact, unique true score variance associated with each of the latent variables specified within a correlated-factor model solution. As will be demonstrated in the simulation below, it is possible that less than all three of the latent variables (A, B, C) may be found to be associated with clear empirical support (i.e., unique true score variance), even when the inter-latent variable correlations between the latent variables are substantially less than 1.0. To overcome the limitations of the correlated-factor model, it is argued in this paper that the higher-order model (Burt, 1950; Rindskopf & Rose, 1988; Thomson, 1951) can be especially useful for the purposes of determining, relatively unambiguously, whether all of the postulated distinct dimensions associated with a correlated-factor model are, in fact, clearly empirically supported representations of distinct or separable dimensions.

As can be seen in Figure 1 (Model 3), the higher-order model is associated with one general factor defined by three first-order factors (A, B, and C). Often unrecognized is that the higher-order model is, typically, associated with orthogonal latent variable terms (Gignac, 2016). In the current example, there are three orthogonal latent variable terms. Specifically, all of the first-order factors are associated with a residual: RA, RB, and RC. The RA, RB, and RC terms represent the true score variance (i.e., not error variance) that was unaccounted for by the general factor. For example, if the A first-order factor were defined by operation span, an *n*-back task, and letter-number sequencing, the A first-order factor residual (RA) would represent the variance common to the three tasks that was not shared with the other indicators in the model. Thus, depending on the nature of the other indicators in the model, the RA term may represent an independent (or residualised of *g*) working memory capacity construct. By contrast, the general factor may be considered a global memory span dimension. For the purposes of the argument advanced in this manuscript, it is not necessary to postulate theoretically the presence of a general or global dimension of any nature. Instead, the general or global factor is simply specified to help estimate the amount of unique variance associated with the hypothesized group-level dimensions. Thus, specific information from a higher-order model is encouraged to be used simply to evaluate a possibly theoretically preferred correlated-factor model.

As a first-order factor's loading onto the general factor increases, the amount of variance associated with a first-order factor's residual will decrease. It is the position of this paper that should a statistically significant residual variance term be observed, then there would be clear evidence to suggest that a particular dimension is, a least partly, unique or separable. Furthermore, a measure of effect size (omega hierarchical subscale; ω_{hs}) should also be consulted for the purposes of evaluating more fully the plausibility of a separable dimension, as discussed further below.

Estimating Specific Factor Variance

Arguably, the most straightforward method to evaluate clearly the plausibility of one or more group-level factors is to estimate the statistical significance associated with first-order factor residuals within a higher-order model. Additionally, the relevant factor loadings should also be in the same direction, in order to support further the plausibility of one or more specific dimensions.

In addition to testing the relevant variance terms for statistical significance, the strength of unique dimensions can be evaluated on a standardized metric known as coefficient omega hierarchical subscale (omegaHS, ω_{hs}; Reise, Bonifay, & Haviland, 2013). As can be seen in formula (1), omegaHS represents a ratio of true score variance to total variance:

$$\omega_{hs} = \frac{\left(\sum_{i=1}^{s1} \lambda_{s1_i}\right)^2}{\left(\sum_{i=1}^{s1} \lambda_{g_i}\right)^2 + \left(\sum_{i=1}^{s1} \lambda_{s1_i}\right)^2 + \left(\sum_{i=1}^{s1} (1 - h_{s1_i}^2)\right)} \quad (1)$$

where λ_{g} corresponds to the general factor loadings, λ_{s1}, $\lambda_{s2} \ldots \lambda_{s}$ correspond to the specific factor loadings, and $(1 - h^2_s)$ represents a specific factor indicator's unique variance. OmegaHS can be estimated from a higher-order model solution, after it has been decomposed via the Schmid-Leiman (Schmid & Leiman, 1957) decomposition procedure[1]. The omegaHS coefficient (ω_{hs}) may be described as unique (or distinct/separate), as it is independent of the general factor associated with a higher-order model

OmegaHS is a standardized index and ranges from 0 to 1.0. Because it is standardized, it has an arguably intuitive interpretation: the proportion of true score variance relative to total variance (i.e., attributable to the relevant lower-level factor) that is unique to the indicators used

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[1] The Schmid-Leiman decomposition involves multiplying the first-order factor loadings by the respective second-order loadings to obtain the associations between the subtests and the general factor. Additionally, by multiplying the first-order factor loadings by the respective first-order factor residual regression weights, the associations between the subtests and the specific factors can be obtained (see Brunner, Nagy, & Wilhelm, 20012 and/or Gignac, 2007 for didactic demonstrations of the decomposition).

to define a particular first-order factor. Additionally, because omega hierarchical is standardized, the strength of specific factors can be compared numerically.[2] By contrast, the variance associated with a specific factor (e.g., first-order factor residual) would very typically be on a scale that is uninterpretable to the typical reader; it would also likely be incomparable across factors.

Reise, Scheines, Widaman, and Haviland (2012) suggested that omegaHS could be considered an indicator of specific (unique) latent variable strength. Values closer to .00 are indicative of a very weak specific latent variable, whereas values closer to 1.0 are indicative of a very strong specific latent variable. Consequently, omegaHS may be viewed as a representation of effect size that is essentially unaffected by sample size. Although there are no widely acknowledged guidelines for interpreting omegaHS values, Gignac and Watkins (2013) suggested that omegaHS values less than .50 likely render composite scores based on those indicators very difficult, if not impossible, to interpret in applied settings, as less than 50% of the variance in the composite scores would be due to the construct of interest. However, within the context of pure research, where effects can be disattenuated for any level of imperfect reliability via latent variable models, a minimum level of omegaHS is probably not necessary to achieve. However, omegaHS values could potentially be interpreted from a relative magnitude perspective. Rodriguez, Reise, and Haviland (2016b) estimated the omegaHS values associated with the 50 multi-factor solutions, based on 50 previously published correlation matrices. In

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[2] The sampling distribution of omega hierarchical has not yet been established, however, a bootstrapped procedure would likely facilitate a valid approach to testing the difference between two omega hierarchical coefficients, statistically.

total, 147 omegaHS values were estimated by Rodriguez et al. (2016a) and reported in their Table 1. Based on our calculations, the 33rd and 66th percentiles corresponded to omegaHS values of .18 and .30. Thus, we propose that relatively small, typical, and relatively large omegaHS values correspond to the following guidelines: relatively small < .20; typical .20 to .30; and relatively large > .30. OmegaHS values less than .10 should probably considered relatively very small, as fewer than 13% of the omegaHS estimates were so small in our analysis.

It will be noted that omegaHS is a measure of effect size, consequently, it is essentially unaffected by sample size. By contrast, tests of the statistical significance of first-order factor residual terms are affected by statistical power. Thus, a test of significance of the residual term and the omegaHS guidelines proposed above may not necessarily agree with each other. For example, a relatively large omegaHS value may be estimated from a factor solution, however, the corresponding first-order factor residual may not be statistically significant. Consequently, it is recommended in this paper that the statistical significance of the first-order factor residual variance should be viewed as minimally necessary evidence to support more fully the plausibility of a separable dimension of cognitive functioning.

Study Purpose

Although the commonly applied test of the difference between a single-factor model and a correlated-factor model may provide evidence suggestive of the plausibility of three or more group-level (specific) factors, it is the position of this paper that additional useful information can be obtained from the analyses described above to help justify more fully interpretations of dimensional distinctness. Consequently, the purpose of this investigation was to demonstrate that well-fitting correlated three-factor models with an inter-latent variable correlations less than 1.0 do not necessarily imply the presence of three or more specific dimensions within the data.

Instead, further analyses should be considered, such as higher-order modeling, ideally complemented with omegaHS, in order to uncover the number and nature of specific, group-level dimensions associated with data.

The current investigation was separated into two studies. In study 1, a simulation was conducted to demonstrate that a correlated-factor model, on its own, yields arguably ambiguous information on the number and nature of the unique dimensions within the data. In study 2, the implications of the simulation were examined across three previously published investigations that endorsed a correlated three-factor model, based on the observation that the correlated-factor model fit better than the single-factor model and/or all of the inter-latent variable correlations were less than 1.0.

Study 1: Simulation

Method

Data Generation

The simulation portion of this investigation was based on the evaluation of the potential plausibility of a correlated three-factor model, as it is a commonly observed model in the literature. Additionally, the correlated three-factor model and the corresponding higher-order model are equivalent models (Bollen, 1989; Reise, 2012). Three correlation matrices were simulated (all N = 500). As can be seen in Table 1, all three correlation matrices consisted of nine indicators: three specified to measure factor A (a1, a2, and a3), three specified to measure factor B (b1, b2, and b3), and three specified to measure factor C (c1, c2, and c3). Across all three correlation matrices, the correlations between indicators from the hypothesized different specific factors were held constant at .20. By contrast, the magnitudes of the correlations between indicators associated with the same hypothesized specific (group-level) factor were manipulated. Specifically, with respect to correlation matrix 1, the a1, a2, and a3 indicators, the b1, b2, and b3 indicators, and the c1, c2, and c3 indicators were specified to inter-correlate at .45 (see Table 1, top). Thus, correlation matrix 1 was specified to be associated with three specific factors. By contrast, with respect to correlation matrix 2, the a1, a2, and 3 and the b1, b2, and b3 indicators were specified to correlate at .45 (see Table 1, middle). Furthermore, the c1, c2, and c3 indicators were specified to correlate only very minimally above .20 (i.e., .21). Thus, only two specific factors resided within correlation matrix 2. Finally, with respect to correlation matrix 3, only the a1, a2, and a3 indicators were specified to correlate at .45 (see Table 1, bottom). Thus, only one specific factor resided within correlation matrix 3.

Model Testing

As can be seen in Figure 1, three models were tested in this investigation: (1) a single-factor model; (2) a correlated three-factor model; and (3) a higher-order model. All three models were tested across all three simulated correlation matrices. It should be noted that when there are only three (or two) group-level factors, the higher-order model and the correlated factor model are equivalent with respect to degrees of freedom and model fit (Bollen, 1989; Reise, 2012). Model close-fit was evaluated via RMSEA ($\leq$ .060) and TLI ($\geq$ .950). As the single-factor model and the correlated three-factor/higher-order model are nested within each other (Rindskopf & Rose, 1988), a chi-square difference test was used to determine whether the correlated three-factor model and the corresponding higher-order model were a better fit to the data (Steiger, Shaprio, & Browne, 1985). OmegaHS was estimated in all cases via formula (1) and the Schmid-Leiman decomposed standardized solution (Brunner et al., 2012; Gignac, 2007), as specified within Omega (Watkins, 2013).[3] Clear evidence in favour of the hypothesized specific dimensions was considered supported on the basis of two pieces of evidence: (1) statistically significant first-order factor residual variances; and (2) consistently directed Schmid-Leiman loadings/coefficients. Additionally, the strength of the distinct dimensions may be interpreted based on the omegaHS guidelines provided above.

All model solutions were estimated via maximum likelihood in Amos 21 (Arbuckle, 2012). With respect to the single-factor model and the correlated-factor model, the latent variables were scaled by fixing the latent variances to 1.0. The higher-order model was scaled/identified such that the second-order general factor variance was fixed to 1.0, and one each of the first-order factor's loadings were fixed to 1.0. The first-order factor residual coefficients (paths from residuals to first-order factors) were fixed to 1.0 to allow for the free estimation and statistical significance testing of the first-order factor residual variances.

Results

With respect to correlation matrix 1, the single-factor model was associated with poor model-fit, $\chi^2(27) = 312.57$, $p < .001$, RMSEA = .146, TLI = .589. As can be seen in Figure 2 (left-hand side), the general factor loadings were all .51. In contrast to the single-factor model, the correlated-factor model and the higher-order model were both found to be associated with perfect model-fit, $\chi^2(24) = .00$, $p = .999$, RMSEA = .000, TLI = 1.039. As can be seen in Figure 2, the correlated-factor model was associated with inter-latent variable correlations that were all

[3] Within most SEM programs, omega can also be estimated via the implied correlation between an equally weighted composite variable (phantom variable) and the corresponding latent variable or residual variance term (see Gignac, 2014, for demonstration).

appreciably less than 1.0 (i.e., .44, $p < .001$; all 95% CI upper-bounds were less than 1.0). Furthermore, the higher-order model was associated with second-order loadings appreciably less than 1.0 (i.e., .67, $p < .001$; 95%CI: .54/.80). Additionally, the first-order factor residual variances were all statistically significant ($S^2_A = .25$, $p < .001$; $S^2_B = .25$, $p < .001$; $S^2_C = .25$, $p < .001$), which supported the plausibility of three group-level dimensions, independent of the inter-latent variable common variance represented by the general factor. Furthermore, all of the Schmid-Leiman decomposed coefficients associated with each factor were in the same direction (i.e., positive). Finally, the omegaHS values based on the Schmid-Leiman decomposed coefficients (see Figure 2, g and s) were all relatively large, $\omega_{hs.A} = .39$; $\omega_{hs.B} = .39$; $\omega_{hs.C} = .39$.

With respect to correlation matrix 2, the single-factor model was associated with poor model-fit, $\chi^2(27) = 181.31$, $p < .001$, RMSEA = .107, TLI = .723. As can be seen in Figure 2 (middle), the general factor loadings were .54 for the factor A and B indicators and .39 for the factor C indicators. In contrast to the single-factor model, the correlated-factor model and the higher-order model were both found to be associated with perfect model-fit, $\chi^2(24) = .00$, $p = .999$, RMSEA = .000, TLI = 1.048. As can be seen in Figure 2, the correlated-factor model was associated with inter-latent variable correlations that were all appreciably less than 1.0 (i.e., .44 to .65, $p < .001$; all 95% CI upper-bounds were less than 1.0). However, the C first-order factor within the higher-order model was associated with a second-order loading near 1.0 (i.e., .98; 95%CI: .83/1.13), which suggested the implausibility of the C group-level dimension, independent of the inter-latent variable common variance represented by the general factor. Correspondingly, the C first-order factor residual variance was not significant statistically, $S^2_C = .01$, $p = .781$. Furthermore, the corresponding C factor omegaHS was considered relatively very small, $\omega_{hs.C} = .020$. By contrast, the A and B first-order factor residual variances were statistically significant, S^2_A = .25, p < .001; S^2_B = .25, p < .001. Furthermore, all of the Schmid-Leiman decomposed coefficients associated with each of the A and B factors were in the same direction (i.e., positive). Finally, the A and B factor omegaHS values were considered relatively large, $\omega_{hs.A}$ = .39; $\omega_{hs.B}$ = .39.

With respect to correlation matrix 3, the single-factor model was associated with poor model-fit, $\chi^2(27)$ = 59.62, p < .001, RMSEA = .049, TLI = .923. As can be seen in Figure 2 (right-hand side), the general factor loadings ranged from .38 to .63. In contrast to the single-factor model, the correlated-factor model and the higher-order model were both found to be associated with perfect model-fit, , $\chi^2(24)$ = .00, p = .999, RMSEA = .000, TLI = 1.064. As can be seen in Figure 2, the correlated-factor model was associated with inter-latent variable correlations that were less than 1.0, however, the correlation between the B and C latent variables was .95 (95%CI: .78/1.15), which suggested the implausibility of either the B and/or C latent variables as unique dimensions. Correspondingly, the B and C first-order factors within the higher-order model were associated with second-order loadings near 1.0 (i.e., .98; 95%CI: .83/1.14), which suggested the implausibility of the B and C group-level dimensions, independent of the inter-latent variable common variance represented by the general factor. Additionally, the B and C first-order factor residual variances were not significant statistically, S^2_B = .01, p = .759 and S^2_C = .01, p = .759. Finally, based on the Schmid-Leiman decomposed coefficients (see Figure 2, g and s), the omegaHS values associated with the B and C factors were both .021, which was considered relatively very small. By contrast, the A factor was associated with a relatively large omegaHS value, $\omega_{hs.A}$ = .39 (S^2_A = .25, p < .001). Finally, all of the Schmid-Leiman decomposed coefficients associated with the A factor were in the same direction (i.e., positive).

Discussion

Based on the results of this simulation, the correlated three-factor model failed to identify correctly only two specific group-level dimensions within correlation matrix 2, despite the fact that the correlated three-factor model was associated with perfect model-fit, statistically significant factor loadings, and inter-latent variable correlations much less than 1.0. By contrast, the higher-order model identified correctly the number of unique group-level dimensions within the data across all three correlation matrices. Thus, the results associated with the simulation suggest that useful information can be obtained from a higher-order model that cannot be obtained from a correlated-factor model. The process of following-up a well-fitting correlated-factor model with omegaHS estimates is in contrast to the commonly recommended and observed practice of simply testing the difference in model-fit between a single-factor model and a correlated-factor model, in order to potentially support the plausibility of the distinct dimensions implied by the correlated-factor model (e.g., Kline, 2011; Brown, 2015; Byrne, 2010; Zeller & Carmines, 1980).

A correlated-factor model with only two group-level factors may be suggested to be an alternative to the correlated three-factor model tested in this simulation, particularly with respect to correlation matrices 2 and 3. However, there can be challenges with testing the plausibility of only two unique dimensions independent of a general factor. For example, when there is limited number of observed variables (e.g., six subtests; three for each unique factor) it is difficult to evaluate fully the possibility of only two unique dimensions, in addition to a general factor, as a higher-order model with only two first-order factors is just-identified when the second-order loadings are constrained to equality (Zinbarg et al., 2007). In the current simulation, a hybrid higher-order model with two first-order factors (A and B) and direct loadings from tests c1 to c3 onto the second-order general factor is a model that could be tested relatively easily, without any unconventional constraints. Further work in this area of the evaluation of two unique factors is encouraged, considering the frequency with which correlated two-factor models are reported in the literature.

Study 2: Examples with Field Data

The simulation above demonstrated the limitations associated with the correlated-factor model with respect to yielding unambiguous interpretations of group-level factors. By contrast, the higher-order model, in conjunction with omegaHS, yielded additional, relatively unambiguous results. To demonstrate the implications of the simulation to field data, the data associated with three published investigations in the area of cognition will be re-analysed in the second part of this paper. Specifically, competing single-factor and correlated-factor models will be tested. The correlated-factor models endorsed by the authors will be confirmed to be associated with acceptable levels of model-fit, in addition to superior model-fit over the single-factor model. However, the re-analyses will be complemented with corresponding higher-order models and the estimation of omegaHS, in order to verify the plausibility of the group-level factors endorsed by the authors in the original publications. All analyses were performed with Amos 21 via maximum likelihood estimation (Arbuckle, 2012). For thoroughness, in addition the unstandardized first-order factor residual variances (S^2), we reported the standardized first-order factor residual variances (zS^2) in this portion of the investigation. The estimation of standard errors for standardized parameter estimates is more complicated than unstandardized estimates (see Jones & Waller, 2013). Consequently, we used the lavaan (Rosseel, 2012) package for *R* (*R* Core Team, 2016) to test the standardized first-order factor residual variances for statistical significance.

Field Example 1: Friedman et al. (2006)

Friedman et al. (2006) examined the association between executive functions (inhibition, updating, shifting) and intelligence (fluid intelligence and crystallised intelligence). Prior to estimating the structural model, Friedman et al (2006) evaluated the executive functions measurement model (N = 234) based on three indicators of inhibition (antisaccade, stop-signal, stroop), three indicators of updating (keep-track, letter-memory, spatial 2-back), and three indicators of shifting (number-letter, color-shape, category switch). The correlated three-factor model was found to be acceptably well-fitting, $\chi^2(24) = 19.86$, $p = .705$, SRMR = .033, RMSEA = .000, CFI = 1.000, TLI = 1.017. By contrast, the competing single-factor model was not well-fitting, $\chi^2(27) = 97.83$, $p < .001$, SRMR = .079, RMSEA = .106, CFI = .808, TLI = .744. Furthermore, the correlated three-factor model fit better than the single-factor model, $\Delta\chi^2(3) = 77.97$, $p < .001$. Finally, as can be seen in Figure 3, all of the inter-latent variable correlations associated with the correlated three-factor model were well below 1.0.

However, based on the corresponding higher-order model, only the updating and shifting first-order factors were associated with statistically significant residual variance terms: $S^2_{\text{inhibition}} = .00$, $p = .780$; $S^2_{\text{updating}} = .01$, $p < .001$; and $S^2_{\text{shifting}} = 10880.55$ $p < .001$. The standardized first-order factor residual variance terms corresponded to: $zS^2_{\text{inhibition}} = -.07$, $p = .781$; $zS^2_{\text{updating}} = .60$, $p < .001$; and $zS^2_{\text{shifting}} = .61$ $p < .001$. Furthermore, the updating and shifting first-order factors were associated with relatively large omegaHS values, $\omega_{\text{hs.updating}} = .37$ and $\omega_{\text{hs.shifting}} = .45$, based on the guidelines specified in the introduction. By contrast, the inhibition first-order factor was associated with an omegaHS value of essentially zero, $\omega_{\text{hs.inhibition}} = .00$. Thus, although the correlated three-factor model suggested the plausibility of three specific dimensions within the data, the higher-order model results corroborated the plausibility of only two specific dimensions, updating and shifting.

Field Example 2: Gray et al. (2017)

Gray et al. (2017) examined the association between the central executive, the visuospatial sketch pad, and the phonological loop in a correlated three-factor model (N = 168). The central executive latent variable was defined by three indicators (*n*-back auditory, *n*-back visual, number updating), the visual-sketch pad was defined by six indicators (visual span running, location span running, visual span, location span, visual-spatial binding, cross-modal binding), and the phonological loop was defined by four indicators (digit span running, phonological binding, digit span, non-word repetition). The correlated three-factor model was found to be reasonably well-fitting, $\chi^2(62) = 87.64$, $p = .018$, SRMR = .062, RMSEA = .050, CFI = .928, TLI = .909. By contrast, the competing single-factor model was not well-fitting, $\chi^2(65) = 117.34$, $p < .001$, SRMR = .071, RMSEA = .069, CFI = .853, TLI = .823. Furthermore, the correlated three-factor model fit better than the single-factor model, $\Delta\chi^2(3) = 29.70$, $p < .001$. Finally, as can be seen in Figure 3, all of the inter-latent variable correlations associated with the correlated three-factor model were well below 1.0.

However, based on the corresponding higher-order model, only the central executive and the phonological loop first-order factors were associated with statistically significant residual variance terms, $S^2_{\text{central executive}} = .002$, $p = .048$; $S^2_{\text{sketch pad}} = -.03$, $p = .642$; $S^2_{\text{phonological loop}} = .24$ $p = .038$. The corresponding standardized first-order factor residual variances were $zS^2_{\text{central executive}} = .52$, $p = .004$, $zS^2_{\text{sketch pad}} = -.17$, $p = .641$, $S^2_{\text{phonological loop}} = .77$, $p < .001$. Furthermore, the phonological loop factor and central executive first-order factors were both associated with relatively large omegaHS values, $\omega_{\text{hs.phonological loop}} = .39$ and $\omega_{\text{hs.central executive}} = .30$, based on the guidelines specified in the introduction (i.e., central executive at the very top-end of typical). By contrast, the sketch pad first-order factor was associated with an omegaHS value of essentially zero, $\omega_{\text{hs.sketch pad}}$ = .00. Thus, although the correlated three-factor model suggested the plausibility of three specific dimensions within the data, the higher-order model results corroborated the plausibility of only two specific dimensions, the central executive and the phonological loop.

Field Example 3: Fluid Intelligence, Working Memory, and Processing Speed

Several highly cited investigations have examined the associations between fluid intelligence, working memory, and processing speed with correlated factor models (e.g., Conway, Cowan, Bunting, Therriault, & Minkoff, 2002; Fry & Hale, 1996). Arguably, whether fluid intelligence, working memory, and processing speed are associated with any statistically significant unique variance remains an open question, as previous investigations did not estimate and/or test the unique true variance associated with each of the hypothesized dimensions. Arguably, previous empirical investigations in the area were not based on particularly large samples ($N < 125$) and/or unrepresentative samples (university students; private school students). Consequently, in this field example, the associations between fluid intelligence, working memory, and processing speed were estimated with the WAIS-IV normative sample correlation matrix for 35- to 44-year-olds ($N = 200$; Wechsler, 2008). Fluid intelligence was defined by Matrix Reasoning, Visual Puzzles, and Figure Weights. Working memory was defined by Digit Span Backwards, Digit Span Sequencing, and Letter-Number Sequencing. Finally, processing speed was defined by Symbol Search, Coding, and Cancellation.

The correlated three-factor model was found to be well-fitting, $\chi^2(24) = 31.12$, $p = .150$, SRMR = .037, RMSEA = .039, CFI = .989, TLI = .984. By contrast, the competing single-factor model was not well-fitting, $\chi^2(27) = 156.69$, $p < .001$, SRMR = .084, RMSEA = .155, CFI = .806, TLI = .742. Furthermore, the correlated three-factor model fit better than the single-factor model, $\Delta\chi^2(3) = 125.57$, $p < .001$. Finally, as can be seen in Figure 4, all of the inter-latent variable correlations associated with the correlated three-factor model were well below 1.0.

Based on the corresponding higher-order model, all three first-order factors were associated with statistically significant residual variance terms, $S^2_{\text{fluid}} = 1.50$, $p = .022$; $S^2_{\text{working memory}} = 1.30$, $p = .009$, $S^2_{\text{processing speed}} = 3.76$, $p < .001$.[4] The corresponding standardized first-order factor residual variances were $zS^2_{\text{fluid}} = .25$, $p = .016$; $zS^2_{\text{working memory}} = .29$, $p = .004$, $zS^2_{\text{processing speed}} = .55$, $p < .001$. Furthermore, the Gf and WM first-order factors were associated with approximately typically sized omegaHS values ($\omega_{\text{hs.fluid}} = .20$; $\omega_{\text{hs.working memory}} = .23$) and the Gs first-order factor was associated with a relatively large omegaHS value ($\omega_{\text{hs.processing speed}} = .41$). Thus, the higher-order model results corroborated the plausibility of all three specific dimensions.

Discussion

Three field data examples were evaluated with respect to the plausibility of the interpreted group-level latent variables associated with the well-fitting correlated-factor models. Only in the third example were all of the group-level factors implied by the correlated factor model results supported by the first-order factor residual variance terms. Thus, although field

[4] As the analyses in this field example were based on correlation matrices with test publisher generated standard deviations of 3.0 (i.e., not raw score standard deviations), the first-order factor residual variances were tested for statistical significance via parametric bootstrapping (Yuan & Hayashi, 2006).

examples 1 and 2 interpreted all of the group-level factors as to some degree distinct, one of the group-level factors associated with each of field examples 1 and 2 was not sufficiently strong as to be distinguishable from the common variance shared by the latent variables.

General Discussion

The results of this investigation suggest that researchers should probably not rely upon the observation of a correlated-factor model that fits better than single-factor model, in combination with the observation of inter-latent variable correlations substantially less than 1.0, as clear evidence to suggest the plausibility of two or more distinct, group-level dimensions. Stated alternatively, researchers should not interpret a correlated-factor model of *p* factors as necessarily evidence of *p* unique dimensions. Instead, further work should be considered, such as testing a higher-order model and estimating omegaHS coefficients, in order to evaluate fully the number and strength of the hypothesized specific dimensions. The effects were demonstrated via simulation and via the re-analysis of previously published correlation matrices.

It should be emphasised that the purpose of this investigation was not to discount the correlated-factor model as a potentially valid representation of data. Some researchers clearly prefer a correlated-factor model over other models on theoretical grounds (e.g., Conway & Kovacs, 2013; Kovacs & Conway, 2016; van der Maas, Dolan, Grasman, Wicherts, Huizenga, & Raijmakers, 2006). Thus, it is acknowledged that factors may inter-correlate positively for reasons other than a general factor. However, arguably, a theoretical preference for a well-fitting correlated-factor model should not be considered sufficient grounds to justify unambiguous interpretations of dimensional distinctness, in the absence of supplementary analyses such as those conducted in this investigation.

Beyond the evaluation of any particular correlation matrix for dimensional distinctness, the results of this investigation have potential implications for the immense proliferation of newly proposed constructs of memory and memory mechanisms (250+) over the years (Tulving, 2007). That is, it is possible that many of the proposed constructs are simply redundant with other previously established memory constructs. Additionally, the validity of many proposed dimensions of memory may rest, in part, on ambiguous evidence derived from correlated-factor models. Arguably, many newly proposed constructs may not be measured, currently, in a manner that can be expected to differentiate themselves sufficiently so as to be associated with statistically significant unique true score variance terms and/or omegaHS values that are respectably large.

Additionally, the relative weakness associated with the specific latent variable unique variances observed in this investigation may help explain some surprising results in the cognitive literature. For example, Mogle, Lovett, Stawski, & Sliwinki (2008) failed to observe a unique effect of working memory capacity on fluid intelligence (β = -.08), whereby Raven’s was regressed onto a correlated four-factor model of processing speed, primary memory, working memory, and secondary memory (N = 383). However, based on a corresponding higher-order model (available upon request), there was no evidence to suggest the plausibility of a unique WMC factor based on these data (WMC first-order factor residual variance = .00).

It will also be noted that the re-analysis of Friedman et al.’s (2006) correlation matrix via the higher-order model reported in this investigation failed to yield evidence in favour of a unique inhibition latent dimensions (omegaHS = .00, p = .780). Correspondingly, Friedman et al. (2008) and Miyake and Friedman (2012) reported the absence of empirical evidence in favour of a distinct inhibition latent variable, based on a bifactor model of executive functions defined by the same measures used in Friedman et al. (2006). Thus, the results of our re-analysis of Friedman et al.'s (2006) correlation matrix corroborated the results reported by Friedman et al. (2008) and Miyake et al. (2012).

Finally, we discuss here, briefly, that failure to observe unambiguous evidence in favor of the plausibility of a distinct dimension may be the result of one or more causes. First, it may be that the hypothesized specific dimension simply does not exist, independently of the general factor. Such a conclusion would have theoretical implications. However, it may also be that the quality of the indicators used to define the latent variable is insufficient. Quality, here, may refer to the psychometric properties of the individual subtests, or the subtest selections made by the researchers to define the latent variable (i.e., insufficiently representative of the dimension of interest). Finally, the number of indicators used to define the latent variable may be insufficient. Researchers often use three indicators to define a latent variable in a multi-factor model. Three indicators should probably be viewed as the absolute minimum, rather than an acceptable or respectable number of indicators to define a latent variable. In this context, it is important to note that simulation work suggests that as many as 10 indicators may be required to identify a unique dimension as statistically significant, in the presence of a relatively strong general factor (Sinharay, 2010). Thus, many hypothesized specific dimensions may not have empirical support, when analysed with the recommended methods described in this investigation. Such an observation may have theoretical implications. However, it may also be a reflection of the quality, nature, and/or number of the tests included in the model.

Limitations

A higher-order model with two or three lower-order factors is statistically equivalent (i.e., same number of degrees of freedom and model-fit) to a correlated-factor model with two or three factors (Bollen, 1989; Reise, 2012). Consequently, a higher-order model was considered the most appropriate modeling approach to help evaluate the distinctness of the hypothesized dimensions. However, it will be noted that it is also possible to estimate omegaHS from a bifactor model solution (e.g., Brunner & Süß, 2005; Gignac & Watkins, 2013; Kretzschmar, Neubert, Wüstenberg, & Greiff, 2016; Reise, 2012). Thus, the current investigation is limited in that it did not evaluate the bifactor model. However, the higher-order model and the bifactor model are very similar in several respects. Gignac (2016) demonstrated that the higher-order model and the bifactor model will yield identical solutions to the degree that the proportionality constraint implied by the higher-order model is consistent with the data. In practice, however, the implied proportionality constraint may not be expected to be satisfied with field data (Beaujean, Parkin, & Parker, 2014; Canivez, 2014; Chen, West, Sousa, 2006; Gignac, 2006; Gignac, 2008; Gignac, 2013; Nelson, Canivez, & Watkins, 2013; Watkins, 2010). Consequently, researchers may consider the bifactor model as a viable model in many cases. Ultimately, researchers interested in evaluating hypotheses relevant to the distinctness of correlated dimensions are encouraged to extend their analyses beyond a correlated-factor model, whether from a higher-order modeling strategy or a bifactor modeling strategy.

Only a small number of simulated correlation matrices were generated in this investigation. Thus, the simulation should not be considered comprehensive. Comparisons between the correlated three-factor model and the typical higher-order model with three first-order factors is especially attractive, as they are equivalent (Bollen, 1989; Reise, 2012). The evaluation of dimensional distinctness in correlated-factor models with more than three hypothesized group-level dimensions cannot be achieved as easily by the strategy employed in this investigation, because correlated factor models with more than three group-level factors are not equivalent to higher-order models with more than three group-level factors (Loehlin, 2004). In practical terms, a correlated-factor model with more than three group-level factors will tend to account for more shared inter-latent variable variance, in comparison to a higher-order model with a general factor. The reason is that a typical higher-order global factor can only account for a single dimension. By contrast, the shared variance between four or more group-level factors associated with a correlated-factor model may be due to two or more dimensions. Despite the above, in cases where there are more than three group-level factors, researchers may be able to uncover the nature of the higher-order multi-dimensionality.

For example, as can be seen in Figure 5, a correlated-factor model with six theorized group-level factors has been specified. All of the group-level factors have been specified to be correlated. A single higher-order factor may not be expected to account for all of the shared variance between the group-level factors. Specifically, in this hypothetical case, there may be common variance shared between the verbal working memory scales and common variance shared between the spatial working memory scales, independent of the global second-order factor. However, a higher-order model, such as that depicted as model 2 (Figure 5), may account for all of the shared variance between the group-level factors. Thus, with respect to such a higher-order model, the first-order factor residuals may be tested for statistical significance and the omegaHS values estimated from the Schmid-Leiman decomposed solution, as per the methods recommended in this investigation.

Finally, the test of the first-order factor residual variances for statistical significance is naturally impacted by statistical power. Arguably, the field study examples reported in this investigation were not especially powerful, as they were associated with sample sizes of approximately 200 or less. However, one of the field study examples (Study 3) yielded statistically significant residual variance terms for all three hypothesized unique dimensions based on a sample size of 200. Furthermore, two of the corresponding omegaHS values were only moderate in magnitude (≈ .20).[5] Thus, a very large sample size may not be required to gain a respectable level of statistical power, in this context. Nonetheless, a comprehensive evaluation of the effects of sample size, inter-indicator correlation sizes, and inter-latent variable correlation sizes on statistical power would be a valuable contribution to the literature.

Conclusion

In the context of multi-dimensional measures, Rodriguez, Reise, and Haviland (2016a) encouraged researchers to "…be aware of the sources of reliable variance and give more serious thought to the nature of narrow-band constructs more generally." (p. 234). Direct evaluations of the plausibility of group-level dimensions occurs only rarely in the context of cognitive research which endorses a correlated-factor model. The area of differential cognitive science is not unique in this respect. It is hoped that the procedures, results, and implications described in this investigation will help encourage researchers evaluate hypothesizes relevant to dimensional distinctness in a manner that includes the testing of residual variances for statistical significance, as well as the estimation and interpretation of omegaHS.

[5] Although omegaHS is not directly affected by sample size, power is a function of sample size, alpha and effect size (Aberson, 2010).

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Table 1

Simulated Correlation Matrices

	a1	a2	a3	b1	b2	b3	c1	c2	c3
Correlation Matrix 1									
a1	1.0								
a2	.45	1.0							
a3	.45	.45	1.0						
b1	.20	.20	.20	1.0					
b2	.20	.20	.20	.45	1.0				
b3	.20	.20	.20	.45	.45	1.0			
c1	.20	.20	.20	.20	.20	.20	1.0		
c2	.20	.20	.20	.20	.20	.20	.45	1.0	
c3	.20	.20	.20	.20	.20	.20	.45	.45	1.0
Correlation Matrix 2									
a1	1.0								
a2	.45	1.0							
a3	.45	.45	1.0						
b1	.20	.20	.20	1.0					
b2	.20	.20	.20	.45	1.0				
b3	.20	.20	.20	.45	.45	1.0			
c1	.20	.20	.20	.20	.20	.20	1.0		
c2	.20	.20	.20	.20	.20	.20	.21	1.0	
c3	.20	.20	.20	.20	.20	.20	.21	.21	1.0
Correlation Matrix 3									
a1	1.0								
a2	.45	1.0							
a3	.45	.45	1.0						
b1	.20	.20	.20	1.0					
b2	.20	.20	.20	.21	1.0				
b3	.20	.20	.20	.21	.21	1.0			
c1	.20	.20	.20	.20	.20	.20	1.0		
c2	.20	.20	.20	.20	.20	.20	.21	1.0	
c3	.20	.20	.20	.20	.20	.20	.21	.21	1.0

Note. N = 500; correlation matrix 1 corresponds to a model with three specific factors (A, B, and C); correlation matrix 2 corresponds to a model with two specific factors (A and B); correlation matrix 3 corresponds to a model with one specific factor (A).

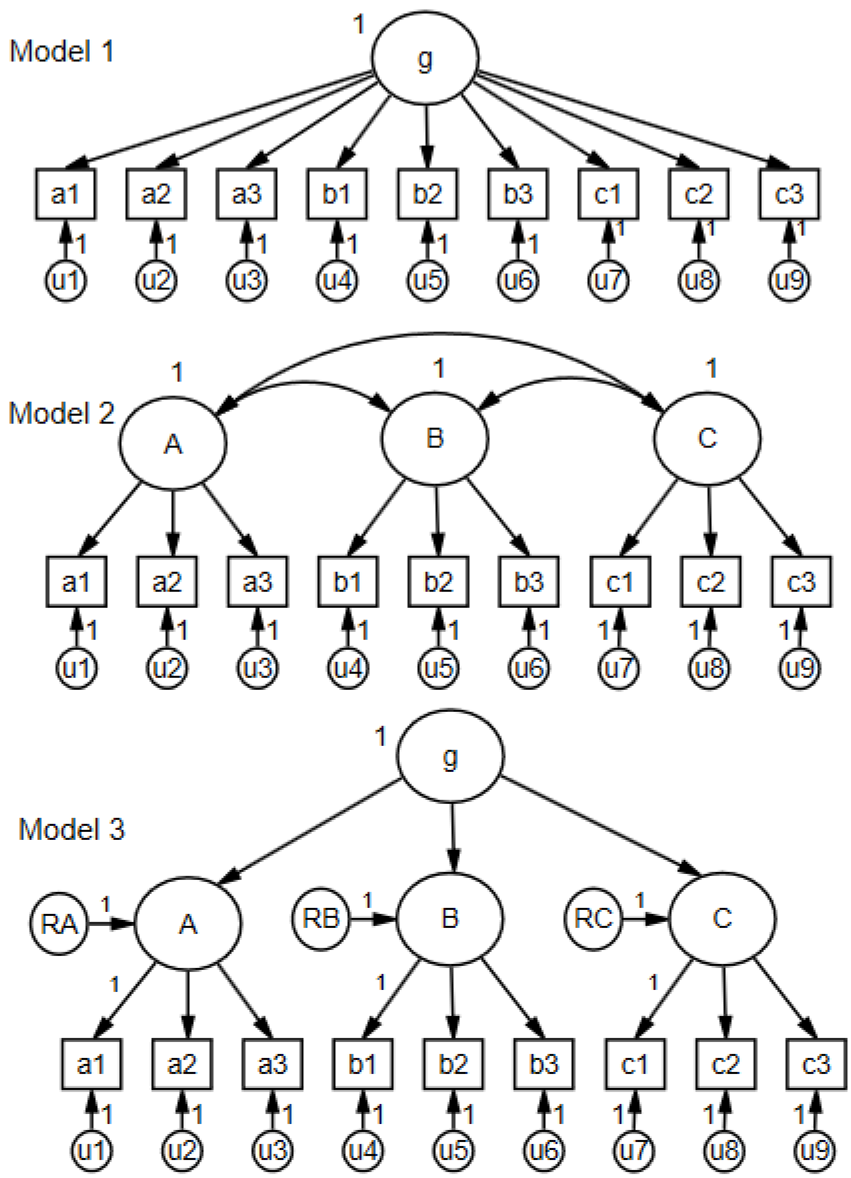

Figure 1. Competing measurement models; Model 1 = single-factor model; Model 2 = correlated factor model; Model 3 = higher-order model.

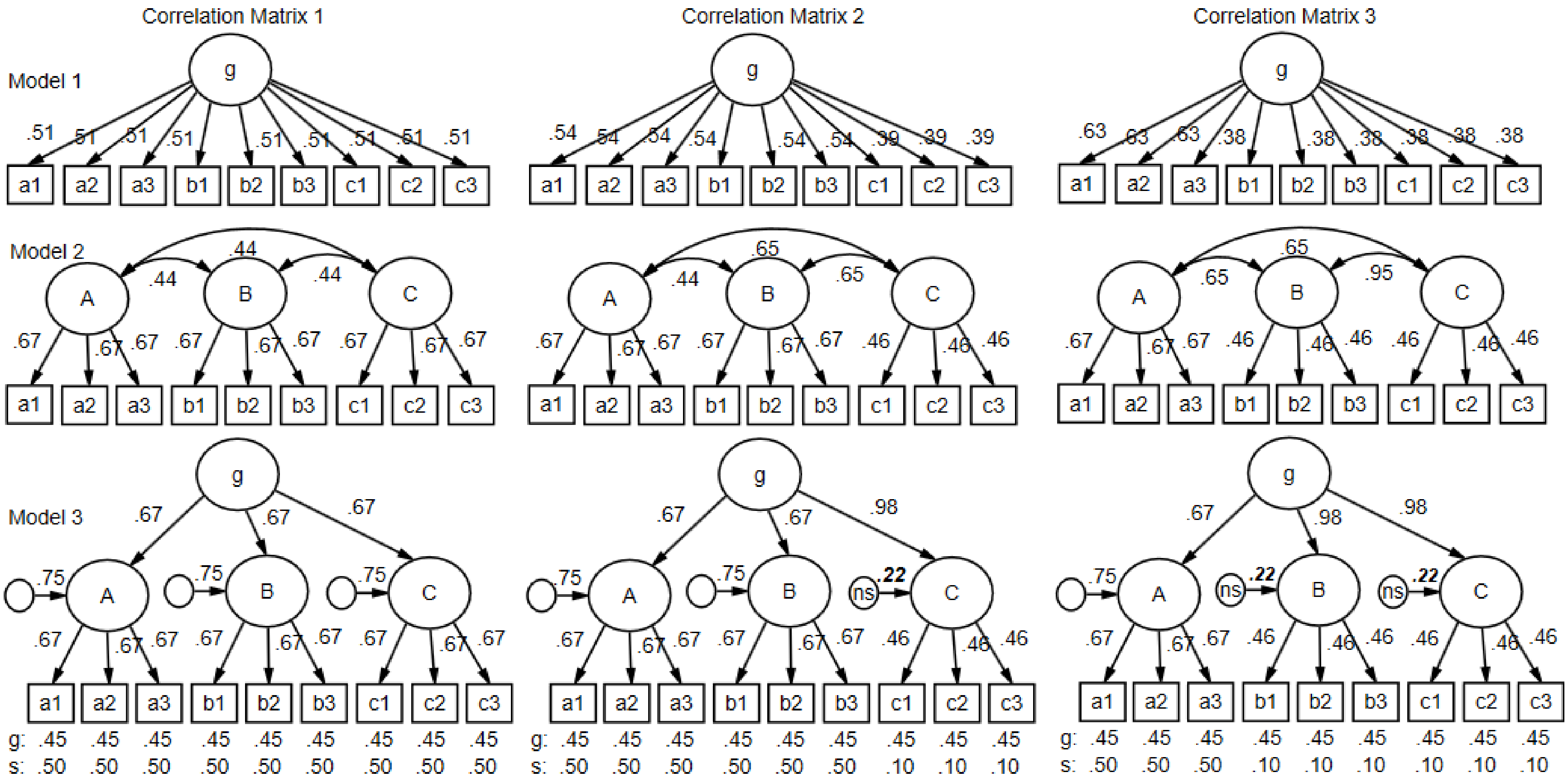

Figure 2. Completely standardized factor solutions associated with the models tested in the simulation; loadings in bold/italic were not significant statistically (p > .05); ns = non-significant first-order factor residual variance term (p > .05); the indicator residuals were omitted for clarity; g and s rows = Schmid-Leiman decomposed effects; the omega estimates (ω), i.e., without the higher-order common variance influence removed, were the following for the three group-level dimensions: correlation matrix 1: ω_A = .71, ω_B = .71, ω_C = .71; correlation matrix 2: ω_A = .71, ω_B = .71, ω_C = .45; correlation matrix 3: ω_A = .71; ω_B = .45, ω_C = .45.

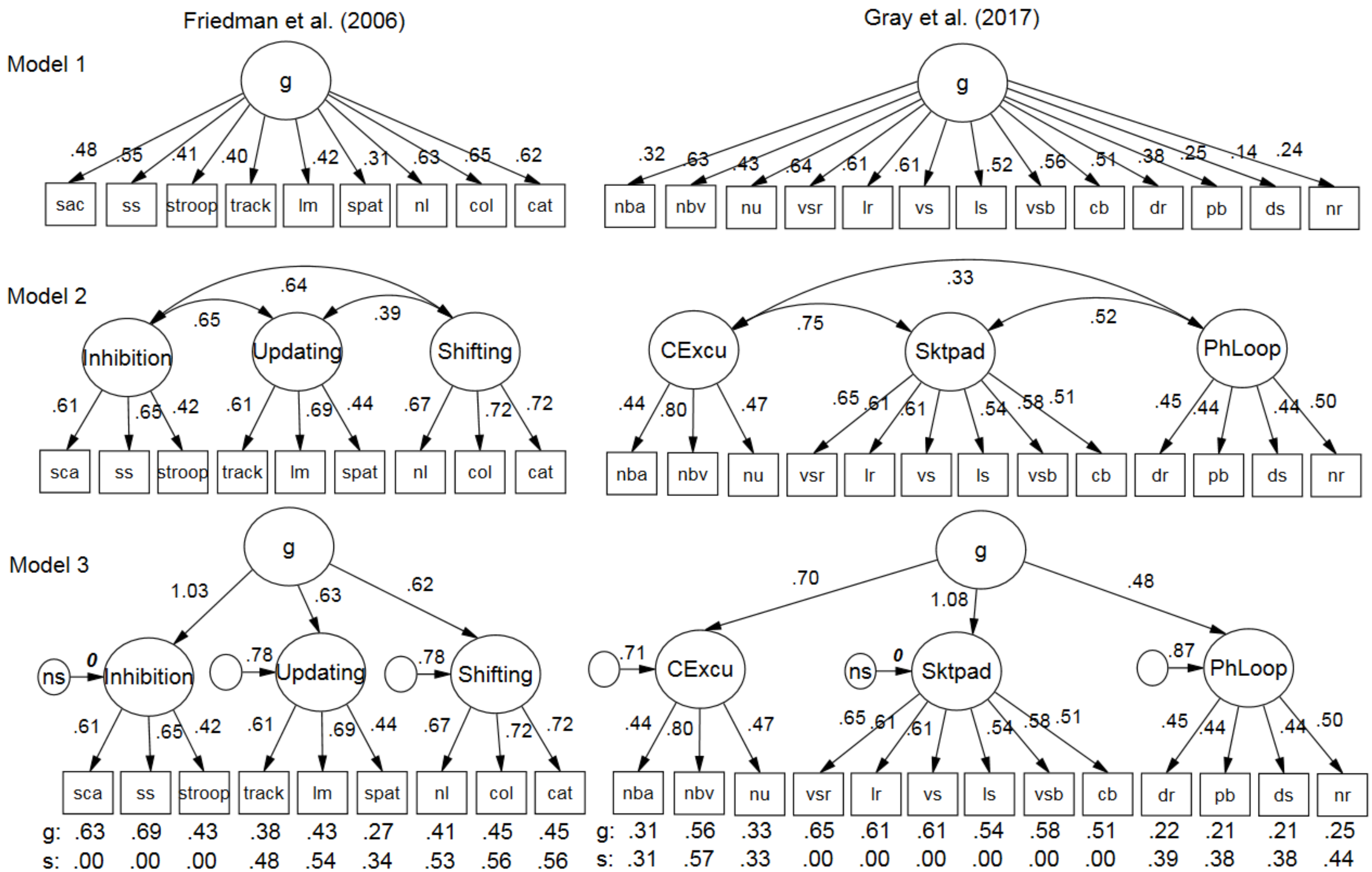

Figure 3. Completely standardized factor solutions associated with the field study re-analyses (study 1 and 2); coefficients in bold/italic were not significant statistically ($p > .05$); ns = non-significant first-order factor residual variance term ($p > .05$); the regression coefficient values specified at 0 are adjustments to reflect the impossibility of negative variance; the indicator residuals were omitted for clarity; g = Schmid-Leiman decomposed general coefficient; s = Schmid-Leiman decomposed specific coefficient; the omega estimates (ω), i.e., without the higher-order common variance influence removed, were the following for the three group-level dimensions: Friedman et al. (2006): $\omega_{inhibition} = .61$, $\omega_{updating} = .61$, $\omega_{shifting} = .74$; Gray et al. (2017): $\omega_{central\ executive} = .59$, $\omega_{sketch\ pad} = .76$, $\omega_{phonological\ loop} = .51$.

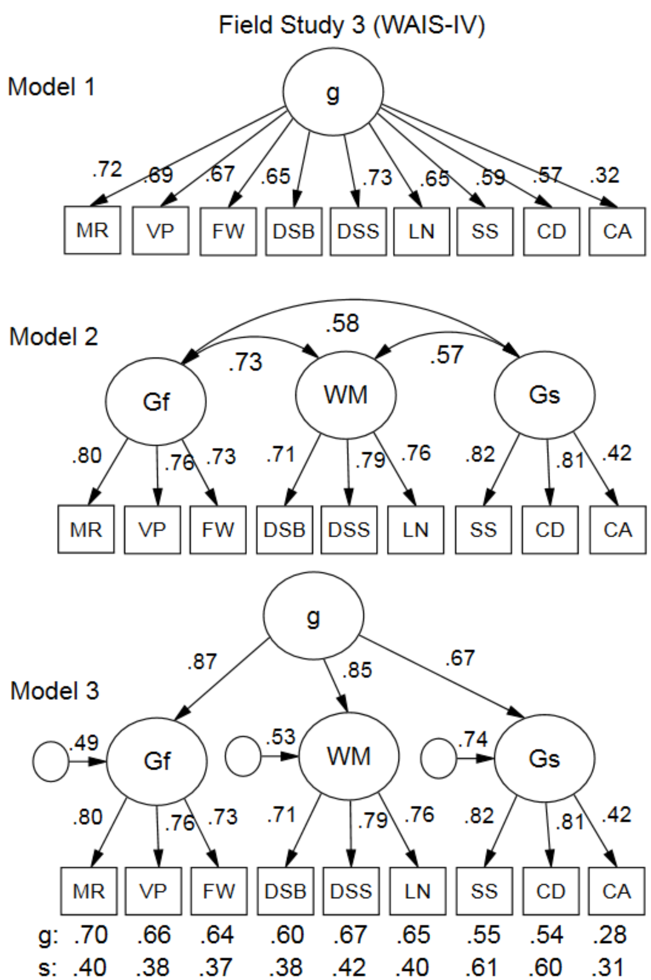

Figure 4. Completely standardized factor solutions associated with the field study re-analyses (study 3); the indicator residuals were omitted for clarity; g = Schmid-Leiman decomposed general coefficient; s = Schmid-Leiman decomposed specific coefficient; the omega estimates (*ω*), i.e., without the higher-order common variance influence removed, were the following for the three group-level dimensions: $\omega_{\text{fluid}} = .81$; $\omega_{\text{working memory}} = .80$, $\omega_{\text{processing speed}} = .74$.

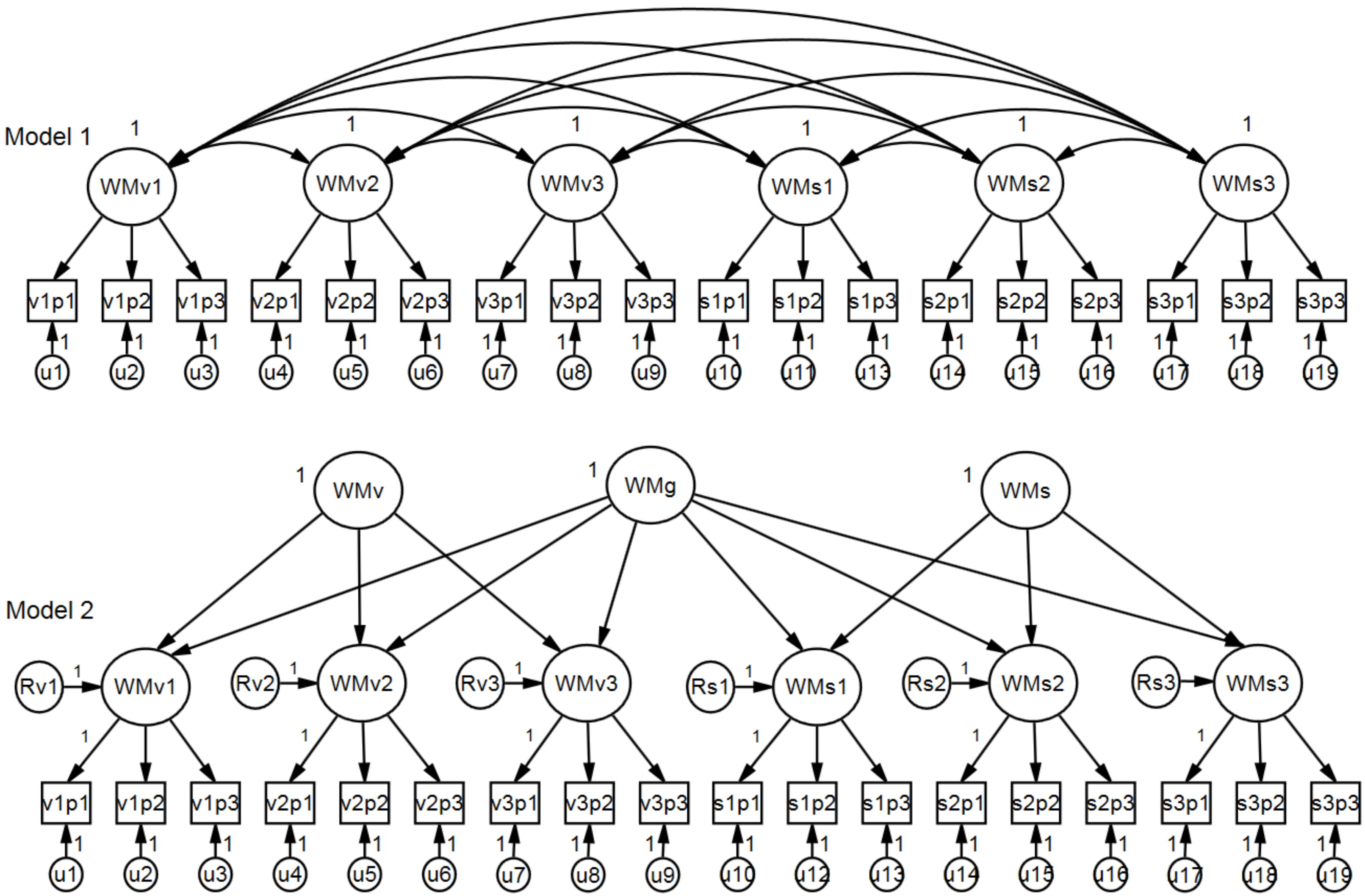

Figure 5. A correlated six-factor model and a hypothetical higher-order model that could potentially be used to help evaluate the number and strength of the specified group-level factors associated with the correlated six-factor model.