At What Sample Size Do Latent Variable Correlations Stabilize?

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Abstract

We conducted a Monte-Carlo simulation within a latent variable framework by varying the following characteristics: population correlation ($\rho = .10, .20, .30, .40, .50, .60, .70, .80, .90, \text{ and } 1.00$) and composite score reliability (coefficient omega: $\omega = .40, .50, .60, .70, .80, \text{ and } .90$). The sample sizes required to estimate stable measurement-error-free correlations were found to approach $N = 490$ for typical research scenarios (population correlation $\rho = .20$; composite score reliability $\omega = .70$) and as high as $N = 1,000+$ for data associated with lower, but still sometimes observed, reliabilities ($\omega = .40$ to $.50$). We encourage researchers to take into consideration reliability, when evaluating the sample sizes required to produce stable measurement-error-free correlations.

Keywords: correlation, accuracy, reliability, sample size, simulation
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Introduction

Researchers commonly conduct research within the frequentist data analytic paradigm (i.e., probability as the long-run expected frequency of occurrence), which involves the specification of a null hypothesis (whether explicitly or implicitly). Data are then analysed statistically to determine the probability with which the null hypothesis can be rejected. Researchers have been encouraged to conduct research with a sufficient amount of statistical power, where statistical power is defined as the probability of rejecting the null hypothesis, when it is in fact false in the population (Cohen, 1988).

Three commonly described characteristics known to influence power include: (1) the significance level alpha ($\alpha$), (2) the magnitude of the effect in the population ($\delta$), and (3) sample size ($N$). In practice, researchers have limited scope to manipulate $\alpha$, i.e., the probability demarcation criterion used to distinguish a non-significant result from a statistically significant result (researchers are essentially fixed on $\alpha = .05$, but see Lakens et al., 2018). Furthermore, researchers cannot influence the size of an effect in the population. By contrast, there is some latitude in the determination of sample size.

In practice, researchers may determine the sample size required to test a hypothesis with a specified level of power. Researchers have widely adopted Cohen’s (1988) minimum power recommendation of .80, which implies that an analysis, based on a sample of data sized $N$, will have an 80% or more chance of rejecting the null hypothesis, if the null hypothesis is in fact false in the population. Consequently, power analyses often involve the determination of the sample size required to test a particular hypothesis, based on $\alpha = .05$ and a specified anticipated effect size of $\delta$. 
Many tables and figures have been published over three decades to help researchers determine the sample sizes required to achieve particular levels of power (e.g., Cohen, 1992; Murphy, Myors, & Wolach, 2012). More recently, a slightly different approach to the evaluation of statistical power, known as the point of stability, has been proposed to help guide researchers in the determination of sample sizes in their research. Schönbrodt and Perugini (2013) described the point of stability as the sample size at which the deviations between a sample estimate and the population parameter are so small (stable) as to be acceptable from a substantive perspective. Thus, a stable sample estimate is one that is close to the population parameter value, and can be expected to remain close to the population parameter value with a specified level of probability (say, 80%, as per statistical power of .80), within the context of repeated sampling with the same sized N.

Using Schönbrodt and Perugini’s (2013) language, stable sample estimate deviations should fluctuate only within a predefined ‘corridor of stability’. Schönbrodt and Perugini’s (2013) suggested that the corridor of stability should have upper and lower bounds equal to |.10|, as a correlation of |.10| is considered small, based on Cohen’s (1992) guidelines. Thus, a key difference between statistical power and the point of stability approach is that the point of stability approach relies upon the corridor of stability to demarcate the acceptable limits of a stable estimate, whereas statistical power relies upon the significance level (\(\alpha\)) to help demarcate a statistically significant effect.

Thus, if an acceptable corridor of stability can be agreed upon (i.e., |.10|), then, as per statistical power sample size requirement calculations, Schönbrodt and Perugini (2013) contended that points of stability could be estimated across variously sized population effects estimated from variously sized N. To estimate points of stability, Schönbrodt and Perugini
(2013) conducted a Monte-Carlo simulation across variously sized population correlations ($\rho = .10$ to $\rho = .70$) estimated from a range of sample sizes ($N = 20$ to $N = 1000$). More specifically, their simulation method involved calculating confidence intervals (80%, 90% and 95%) for 100,000 bootstrapped points of stability to identify the minimum sample size for each correlation. Based on the results of their simulation, as well as balancing accuracy and confidence, Schönbrodt and Perugini (2013) suggested that sample sizes approaching 240 to 250 may be minimally appropriate for typical differential psychology research scenarios (i.e., $r \approx .20$).1 Although Schönbrodt and Perugini’s (2013) simulation provided insight into the stability of correlations for observed scores, their results may not extend to correlations disattenuated for imperfect reliability, such as latent variable correlations.

**Correlations and Reliability of Scores**

A factor known to impact the magnitude of a correlation between observed scores is the reliability associated with the observed scores (Nunnally & Bernstein, 1994). Scores with higher levels of reliability yield larger effect sizes, all other things equal. Stated alternatively, the correlation between two observed variables will be smaller than the theoretical true score correlation to the degree that the observed variable score reliabilities are less than 1.0 (see $r_{\text{max}}$; Nunnally & Bernstein, 1994). Consequently, in addition to sample size, significance level ($\alpha$), and magnitude of effect size in the population, reliability is sometimes mentioned as a factor that

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2 The treatment, here, is circumscribed to correlations. However, the impact of reliability would be expected to extend to other effect sizes (e.g., Cohen’s $d$; Bobko, Roth & Bobko, 2001).
can impact the power of a statistical test (Beckstead, 2013; Cole & Preacher, 2014; Cleary & Linn, 1969; Goodwin & Leech, 2006; Sutcliffe, 1958)\(^3\).

Methods have been developed to estimate correlations not impacted by the attenuating influence of imperfect reliability. For example, there is the classical test theory disattenuation formula, which involves dividing the point estimate correlation by the square root of the product of the reliabilities (e.g., Cronbach’s coefficient alpha; or McDonald’s coefficient omega) of the composite scores (Nunnally & Bernstein, 1994; Spearman, 1904). Such a procedure yields a true score correlation, i.e., a correlation disattenuated of the effects of measurement error. Additionally, correlations estimated within a latent variable framework are also known to represent measurement-error-free correlations (Jöreskog & Sörbom, 1989; Loehlin, 1992). In fact, true score correlations estimated via the classical test theory disattenuation formula and measurement-error-free correlations estimated within a latent variable framework will be equal, to the extent that the assumption of essential tau-equivalence (i.e., equal factor loadings across all observed indicators of a latent variable) is satisfied (Fan, 2003).

Notwithstanding the rigorousness with which classical test theory and latent variable modeling have been established, some have urged caution with respect to the estimation of latent variable effects (e.g., correlations, beta-weights, and second-order factor loadings), when the latent variables are defined by indicators with relatively small loadings and/or a small number of indicators (e.g., Bedeian, Day, & Kelloway, 1997; Gignac, 2007) – characteristics known to impact the reliability associated with the latent variables/scores (Hancock & Mueller, 2001). In

\(^3\) We acknowledge the complications associated with this issue in the context of reliability, measurement error, and change scores (see Williams, Zimmerman & Zumbo, 1995)
simple terms, the concern is predicated upon the possibility that effects disattenuated substantially due to low levels of internal consistency reliability may be associated with substantial sampling variability (i.e., reported disattenuated correlations may fluctuate substantially from sample to sample). Correspondingly, Cohen, Cohen, Teresi, Marchi, and Velez (1990) encouraged researchers to at least report the internal consistency reliability associated with the composite scores that correspond to the latent variables in a model to evaluate the magnitude of the influence of the disattenuation effects.

To-date, however, the impact of composite score internal consistency reliability on the stability of estimated measurement-error-free correlations has not been investigated. As described above, Schönbrodt and Perugini’s (2013) investigation was based on single scores (i.e., not composite scores). Therefore, internal consistency reliability was not an issue. However, in practice, researchers often use imperfectly reliable composite scores in their research. Furthermore, some then disattenuate the effects via the classical test theory approach, or they use latent variable modeling to obtain measurement-error-free estimates of effects size. Consequently, in light of previously expressed concerns (Bedeian et al., 1997; Cohen et al., 1990; Gignac, 2007), the purpose of this investigation was to extend the Monte-Carlo simulation research conducted by Schönbrodt and Perugini (2013). Specifically, in addition to varying population effect size and sample size, we manipulated the degree of internal consistency associated with the composite scores.

**Simulation Method**

Extending Schönbrodt and Perugini’s (2013) approach, we calculated the point of stability for specific combinations of a population correlation and internal consistency
reliabilities (omega, ω; McDonald, 1999). The point of stability is defined as the minimum sample size for which sample correlations fluctuate only within a tolerable interval (corridor of stability) around the population correlation. To ensure comparability with previous research, we followed the procedure of Schönbrodt and Perugini (2013, 2018) as closely as possible. Thus, the following steps were performed for the Monte-Carlo simulation within the latent variable framework.

(1) Define correlation matrices according to the measurement model displayed in Figure 1. Each correlation matrix is the result of a specific combination of three variables: the population correlation (ρ) between two latent factors, the reliability of the first latent factor (ω₁), and the reliability of the second latent factor (ω₂). Ten different population correlations (ρs = .1, .2, .3, .4, .5, .6, .7, .8, .9, 1.0) and seven different internal consistency reliabilities (ωs = .4, .5, .6, .7, .8, .9, 1.0) were used. As some of

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4 McDonald’s coefficient omega is an alternative approach to the estimation of internal consistency reliability. Importantly, whereas the well-known coefficient alpha has been shown to be biased downwardly, to the extent that the tau-equivalence assumption is not met (i.e., equal factor loadings), coefficient omega is relatively unbiased in that respect (for a summary, see Dunn, Baguley, & Brunsden, 2013). In practice, coefficient omega is calculated on the basis of a factor’s (or latent variable’s) factor loadings. All other things equal (e.g., number of items/indicators), larger factor loadings lead to larger internal consistency reliabilities. Coefficient omega can be calculated by hand (for an explanation and demonstration, see Brunner, Nagy, & Wilhelm, 2012) or more efficiently with freely available statistical software (e.g., the R package psych (Revelle, 2018); or stand-alone program, (Watkins, 2013)).
the matrices were redundant (e.g., \(\omega_1 = 0.4 / \omega_2 = 0.9\), and \(\omega_1 = 0.9 / \omega_2 = 0.4\)), 280 different correlations matrices were defined. The correlations matrices and the corresponding measurement models are available in the online supplement.

(2) Simulate a Gaussian distribution with 1,000,000 cases for each correlation matrix (population data). The data was simulated using Ruscio and Kaczetow’s (2008) method.

(3) Draw \(B = 100,000\) bootstrap samples with \(n_{\text{max}} = 1,500\) cases from the population data.

(4) For each bootstrap sample, calculate the latent correlation based on the measurement model displayed in Figure 1 for different sample sizes \(n\). Starting from \(n_{\text{min}} = 20\), the sample size was increased up to \(n_{\text{max}} = 1,500\) with a step size of 10 (trajectory of the correlation).

(5) Calculate the point of stability for each bootstrapped trajectory. That is, trace back the trajectory from \(n_{\text{max}}\), until it breaks the corridor of stability for the first time. The point of stability for each trajectory is the sample size of this break. We followed Schönbrodt and Perugini’s (2018) updated approach to define the corridor of stability (i.e., the corridor of stability is defined as \(\rho \pm w\) without additional transformations) and used a width \((w)\) of 0.10.

(6) Based on the distribution of 100,000 point of stability values for each reliability-correlation combination, three percentiles (80%, 90%, and 95%) were calculated representing the level of confidence of the point of stability.

Finally, we ran the simulation in the \(R\) environment for statistical computing (\(R\) Core Team, 2018) using RStudio (RStudio Team, 2016) and the packages data.table (Dowle &

The simulation code can be found in the online supplement (https://osf.io/k2dwe/?view_only=79271c5e47134f539ed2788432d1fa01). Based on the simulation code, further results not presented here can be calculated (e.g., regarding a larger corridor of stability based on a width of .20).

Results

For the sake of clarity, only the results of those conditions where the latent factors have an equal degree of internal consistency reliability are presented in detail here. Table 1 displays the findings regarding a level of confidence of .80. In line with Schönbrodt and Perugini’s (2013, 2018) results, the required sample size to achieve a stable correlation point-estimate increased with smaller population correlations. For example, with perfect reliability ($\omega = 1.0$) of both latent factors and a population correlation of $\rho = .20$, correlation point-estimates became stable with a sample size of 220. By comparison, stable correlation point-estimates were achieved with a sample size of 50 for population correlations equal to or greater than .70.

Importantly, however, the results based on imperfectly reliable composite scores showed that the required sample size to estimate a stable true score correlation increased as the internal consistency reliability decreased (see Table 1 and Figure 2). For example, a population correlation of .20 required a sample of $N = 490$ for the reliability $\omega = .70$ condition, a value more than twice as large as the $N = 220$ observed for the corresponding perfect reliability condition reported above. Furthermore, the required sample size increased to $N = 1340$ for the reliability $\omega = .40$ condition (again, $\rho = .20$).
The findings regarding additional levels confidence (i.e., 90% and 95%) are reported in detail in the online supplement (Table S1, 90% confidence interval; Table S2, 95% confidence interval). Briefly, as would be expected, a higher level of confidence increased the required sample size. For example, a sample size of \( N = 930 \) was found to be required for a population correlation of .20 and a reliability of \( \omega = .70 \), in the 95% confidence condition. An even lower reliability (e.g., \( \omega = .40 \)) lead to a required sample size greater than the maximal sample size considered in the present simulation (\( n_{\text{max}} = 1,500 \)). The complete results regarding all combinations of the latent factor reliabilities (i.e., latent factors with unequal degrees of internal consistency reliability) for each level of confidence can be found in the online supplement.

**Discussion**

Much work has been published to establish the nature of statistical power, its importance in the context of the cumulative growth of science, as well as practical examples and guides for its application. Schönbrodt and Perugini’s (2013) introduction of the corridor of stability, in combination with their Monte-Carlo simulation, helped extend this statistical research in a useful way, with respect to observed score correlations. We essentially replicated their key result, based on our \( \omega = 1.0 \) latent variable model Monte-Carlo simulation condition. Specifically, a sample size of approximately 220 was found to be necessary to achieve a stable result (with \( w = .10; \) confidence 80%), for a typically reported observed score correlation in differentially psychology (i.e., \( r \approx .20 \)).

However, in practice, researchers often report true score correlations, based on the classical test theory disattenuation formula, or measurement-error-free correlations, based on latent variable modeling. Our simulation results suggest that sample sizes substantially larger than \( N = 220 \) are required to achieve a stable result (again, with \( w = .10; \) confidence 80%), when
the composite scores are associated with internal consistency reliability of .70, i.e., the minimum recommended for basic research (Nunnally & Bernstein, 1994). Internal consistency reliabilities of approximately .70 are also commonly observed in published studies (Peterson, 1994; Shepperd, Emanuel, Dodd, & Logan, 2016). Thus, based on this investigation’s simulation results, sample sizes closer to \( N = 490 \) may be suggested to be minimally required for a substantial amount of differential psychology type research with composite scores and a level of confidence of .80; and sample sizes up to \( N = 930 \), when aiming for a level of confidence equal to 95%.

In simple terms, lower levels of reliability (i.e., latent variables with smaller factor loadings) appear to yield larger measurement-error-free (true score) correlation standard errors. Consequently, such an effect must be countered by having larger sample sizes, in order to maintain a certain level of power (i.e., keep the standard error from increasing). Interestingly, based on our simulation, we note that the required sample size to achieve stable measurement-error-free correlation estimates appears to increase in a curvilinear fashion with decreasing reliability (see Figure 2). That is, controlling for sample size, more substantial loses in effect size stability were found with decreases in reliability from .50 to .40, in comparison to decreases from .90 to .80. Stated alternatively, the measurement-error-free (true score) correlation standard error appears to increase non-proportionally with decreasing internal consistency reliability (i.e., weaker latent variables).

It should be noted that there is some research to suggest that the reliabilities of composite scores derived from some commonly used measures in differential psychology may be substantially less than .70 (e.g., executive functioning; see Hedge, Powell, & Sumner, 2018, for example). Furthermore, based on a reliability generalization investigation of the Big Five
dimensions of personality, Viswesvaran and Ones (2000) reported that approximately 25% of internal consistency reliability coefficients for Openness and Agreeableness scales were less than .70. Additionally, a quantitative survey of 50 latent variable differential psychology studies (many personality type dimensions) found many weak latent variables, as estimated via coefficient omega hierarchical and coefficient omega specific ($\omega = .20$ to $.30$; Rodriguez, Reise, & Haviland, 2016). Finally, on the basis of a meta-analysis of meta-analyses in differential psychology, Gignac and Szodorai (2016) reported that approximately one third of true score correlations reported in the literature were smaller than $| .20 |$. Thus, in practical terms, for a non-negligible percentage of personality studies, the sample sizes required to achieve stable measurement-error-free estimates of effect size may be expected to be closer to $N = 700$ or more, rather than the sample sizes of approximately $N = 200$ often observed in the latent variable/disattenuated effects literature.

**Limitations & Future Research**

We examined only the bivariate association between two latent variables defined by three indicators. Thus, it remains to be determined the degree to which our results generalize to more complex measurement models (e.g., two or more latent variables with four or more indicators and/or cross-loadings) commonly observed in personality psychology (e.g., Hopwood & Donnellan, 2010). We speculate that, in such more complex models, the sampling variability of the internal consistency reliability estimates might be higher, which would suggest that the sample size recommendations reported in this investigation may be lower-bound estimates for the area of differential psychology (see also Hirschfeld, Brachel, & Thielisch, 2014). More simulation research with more complex models is encouraged.
We also note that we used step size of 10, due to computational time considerations, which likely impacted the accuracy of the results to some degree (e.g., the required sample size was unexpectedly the same for the population correlations of .30 and .40, given a reliability of .40). A smaller step size may yield more precise estimates. In addition, floor and ceiling effects regarding the required sample size were observed (e.g., reliabilities ≥ .70 did not impact the point of stability for the 1.00 population correlation; see Figure 2) implicating that future simulations studies should extend the lower and upper bounds of the sample sizes.

Finally, our investigation did not include a formal analytical treatment of bivariate correlations, reliability, and standard error. Some analytical work has been published on the estimation of standard errors and confidence intervals for true score correlations from a frequentist perspective (Bobko & Rieck, 1980). The degree to which the results reported in this investigation can be integrated with the analytical work in the area remains to be determined.

\footnote{We re-ran the simulation based on the classical test theory disattenuation formula and a step size of 1. In general, the results were similar between both approaches (i.e., the average difference between the two approaches regarding the required sample sizes was \( n_{\text{diff}} = 14 \), with larger sample sizes using the disattenuation formula approach), but the smaller step size provided a finer differentiation with regard to the sample sizes. However, notable differences between both approaches were found for a very low reliability (i.e., \( \omega = .40 \); the average difference between both approaches was \( n_{\text{diff}} = 51 \), with larger sample sizes using the latent variable framework approach). The results based on the classical test theory disattenuation formula are available in the online supplement.}
Conclusion

It is well established that reported effects based on larger sample sizes tend to be more stable. Based on the results of this investigation, the impact of composite score reliability on the stability of measurement-error-free effects are, now, also demonstrated. Consequently, we share previously published concerns (e.g., Bedeian et al., 1997) about latent variable effects, based on one or more latent variables associated with low levels of corresponding composite score reliability. Therefore, as per Cohen et al. (1990), we encourage authors to report the composite score reliabilities (i.e., coefficient omega) that correspond to all latent variables within latent variable models, to help readers gain an appreciation for the stability of the reported measurement-error-free effects.
References


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Table 1

**Required Sample Sizes (Point of Stability) for Different Population Correlations (\(\rho\)) and Equal Internal Consistency Reliabilities (\(\omega\)) of Both Latent Factors: Corridor of Stability Width (\(w\)) = .10 and a Level of Confidence of .80.**

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*Note. \(\omega\) = omega (internal consistency reliability); \(\rho\) = population correlation.*
Figure 1. Measurement model for the Monte-Carlo simulation. The latent factors $\eta_1$ and $\eta_2$ are each defined by three indicators with factor loadings depending on a specified combination of internal consistency reliabilities and a latent (population) correlation $\rho$ between $\eta_1$ and $\eta_2$ (see step 1 of the simulation procedure).
Figure 2. Plot of required sample sizes to achieve adequate estimation stability across magnitude of population correlation size and equal degrees of reliability in composite scores; given a corridor of stability width of $w = .10$ and a level of confidence of 80%.